A Heuristic Study on Kansai Marshall Probe

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1 Introduction

The eddy-current probe for detecting defects in the steam generator tube is placed inside the tube as shown in Fig.1.

Several years ago, the late Lord Marshall of Goring made a proposal to The Kansai Electric Power Company to develop a new type of eddy-current probe now known as “Kansai Marshall Probe (KM Probe)” which can effectively detect circumferential defects of the tube. He also suggested the concept for the new probe. The probe consists of several “E”-shaped ferrite cores (piece blocks), a pair of exciting coils and built-in detecting coils. The exciting coils induce a high-frequency primary field in the tube. Defects in the tube perturb this field creating a secondary field which the detecting coils pick up.

The prototype of the KM probe is shown in Fig.2. The cores are arranged radially around the tube axis so that all the core legs directly face the tube wall. One of the two exciting coils is wound to form a hoop in the space between the upper and middle legs, while the other is placed in between the lower and middle legs. Each of the detecting coils is wound around the respective middle leg.

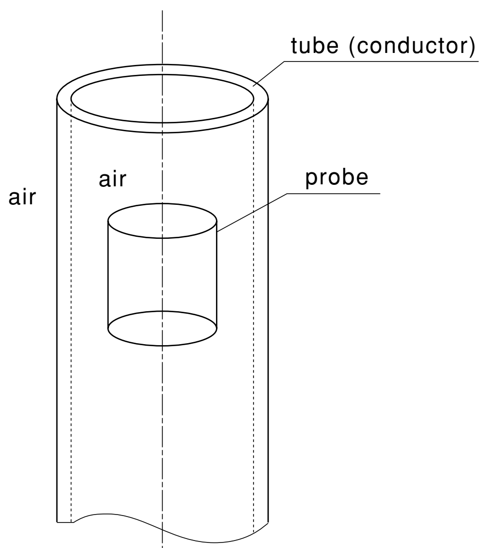

Fig. 1 Steam Generator Tube and ECT Probe

Mr.Yamaguchi(Senior Researcher) has been trying to optimize the design of the probe by means of electromagnetic field computer analyses. Experimental study is also being conducted.

Lord Marshall also suggested that an analytical solution to the problem was necessary before a patent application could be made. He had been working, to the end of his life, on deriving a theoretical solution. A theoretical study is also being conducted by the INSS following his suggestion. The latter is described in this paper.

2 First principle of the Kansai Marshall probe

To enable the KM probe to detect circumferential defects, the primary electric field has an axial component E_z^{prim}. Since the axial eddy current due to E_z^{prim} is blocked by the circumferential defect, the secondary electric field induced by the defect must have an axial component just opposite to E_z^{prim} ; i.e. $E_z^{sec} = -E_z^{prim}$. Now, if this secondary electric field is accompanied by the radial component of magnetic field H_r^{sec} (condition 1), then the secondary field due to circumferential defect is detectable. This is because the radial magnetic field penetrates through the boundary surface to enter the probe space.

Similarly, the axial defect is detectable if the

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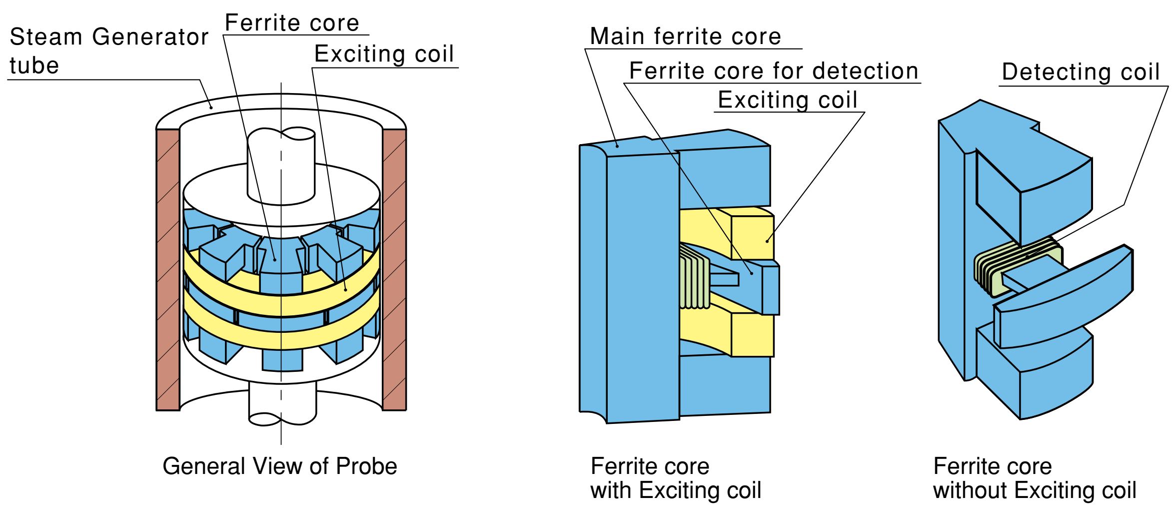

Fig. 2 Kansai Marchall Probe (Courtesy of Mr. Yamaguchi)

primary electric field's circumferential component E_θ^{*prim} is accompanied by the radial magnetic component H_r^{*sec} (condition 2).

In the present study, the two derived conditions described above are assumed to be the first principle of the KM probe.

We began the heuristic study on this principle of the KM probe with the following Maxwell's equations:

$$\vec{\nabla} \times \vec{E} = -i\omega\mu\vec{H}, \tag{1}$$

$$\vec{\nabla} \times \vec{H} = (\sigma + i\omega\varepsilon)\vec{E}, \tag{2}$$

$$\vec{\nabla} \cdot \vec{E} = 0, \tag{3}$$

$$\vec{\nabla} \cdot \vec{H} = 0. \tag{4}$$

which hold in all of the regions of Fig.1 excluding the probe region. In the probe region, where there are ferrite cores, exciting coils and detecting coils, it is hard to write down Maxwell's equations. Let the probe be surrounded by a hypothetical closed surface as shown in Fig.1. The probe produces an electromagnetic field in the whole space outside the probe-space through the boundary conditions (the continuity of the field components) at its surface; therefore the role of the probe is to produce the primary field at this boundary surface. The present heuristic study is based on both the uniqueness of solution in electrodynamics and on the completeness of series expansion in terms of a set of orthogonal functions.

It is assumed that the electromagnetic field depends on time in the form $e^{i\omega t}$. Accordingly, the partial differentiation with respect to time is replaced with factor $i\omega$.

3 Potential functions

3.1 Vector potential $\vec{A}$ for magnetic field

According to Eq.(4), the magnetic field is expressed in terms of the vector potential $\vec{A}$, which is defined by

$$\mu\vec{H} = \vec{\nabla} \times \vec{A}. \tag{5}$$

Substituting Eq.(5) into Eq.(1) and introducing the associated scalar potential ϕ, we obtained the following expression for the electric field:

$$\vec{E} = -i\omega\vec{A} - \vec{\nabla}\phi. \tag{6}$$

Assuming the Lorentz gauge:

$$\vec{\nabla} \cdot \vec{A} + \mu(\sigma + i\omega\varepsilon)\phi = 0, \tag{7}$$

it follows that the potential functions $\vec{A}$ and ϕ respectively satisfy the following equations:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} - \vec{\nabla}(\vec{\nabla} \cdot \vec{A})$$
$$+ i\omega\mu\,(\sigma + i\omega\varepsilon)\vec{A} = 0, \tag{8}$$
$$\nabla^2\phi - i\omega\mu\,(\sigma + i\omega\varepsilon)\phi = 0. \tag{9}$$

3.2 Vector potential $\vec{A^*}$ for electric field

If we pay attention to Eq.(3), instead of Eq.(4), then we can express the electric field in terms of the vector potential $\vec{A^*}$. That is

$$\vec{E^*} = \vec{\nabla} \times \vec{A^*}. \tag{10}$$

In this case, the magnetic field is expressed in terms of the vector potential and the associated scalar potential ϕ^*;

$$\vec{H^*} = (\sigma + i\omega\varepsilon)\vec{A^*} - \vec{\nabla}\phi^*. \tag{11}$$

If we let the Lorentz gauge be

$$\vec{\nabla} \cdot \vec{A^*} - i\omega\mu\phi^* = 0, \tag{12}$$

then the equations for the potential functions become

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A^*} - \vec{\nabla}(\vec{\nabla} \cdot \vec{A^*})$$
$$+ i\omega\mu(\sigma + i\omega\varepsilon)\vec{A^*} = 0, \tag{13}$$
$$\nabla^2\phi^* - i\omega\mu(\sigma + i\omega\varepsilon)\phi^* = 0. \tag{14}$$

We can use either sets ($\vec{A}$, ϕ) or ($\vec{A^*}$, ϕ^*), because these sets are complete by themselves. However, if we impose an additional condition (constraint) on the potential functions, then the set becomes incomplete and we need to use a superposition of the two sets as will be described later.

3.3 Potential functions in Cylindrical coordinates

In cylindrical coordinates r, θ, z, the axial component A_z of the vector potential is independent of the other components A_r and A_θ. Similarly A_z^* is free from the other components A_r^* and A_θ^*. Therefore, we assume that

$$A_z \neq 0, \tag{15}$$
$$A_r = A_\theta = 0, \tag{16}$$

and that

$$A_z^* \neq 0, \tag{17}$$
$$A_r^* = A_\theta^* = 0. \tag{18}$$

According to experiences in the field of engineering electro-dynamics, with a combination of these two types of potential functions, we can describe what we are particularly interested in.

4 Fourier-Bessel expression

4.1 ($\vec{A}$, ϕ) field

The potential functions are assumed to be expressed by the Fourier transform:

$$F(r,\theta,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikz}F(r,\theta,k)dk. \tag{19}$$

With respect to the coordinate θ the potentials are expanded into the Fourier series:

$$F(r,\theta,z) = \sum_{n=0}^{1,2,..} F(r,n,k)e^{in\theta}. \tag{20}$$

Then the differential operators $\frac{\partial}{\partial r}$, $\frac{\partial}{\partial \theta}$, and $\frac{\partial}{\partial z}$ are replaced by $\frac{d}{dr}$, in, and $-ik$, respectively. For simplicity, let the total differentiation with respect to the radial coordinate r be represented by "$'$". Thus the equations (8) and (9) for determining the potential functions of ($\vec{A}$, ϕ) field become

$$A_z'' + \frac{1}{r}A_z' + (\beta^2 - \frac{n^2}{r^2})A_z = 0, \tag{21}$$
$$\beta^2 = -k^2 - i\omega\mu(\sigma + i\omega\varepsilon), \tag{22}$$
$$\phi = \frac{ik}{\mu(\sigma + i\omega\varepsilon)}A_z. \tag{23}$$

Eq.(21) is the Bessel's differential equation, its solution being the cylindrical function $Z_n(\beta r)$. Accordingly Eq.(20) is a Fourier-Bessel series. Hereafter, we deal with the general term (the n-th term) of the series: The general term of A_z is written as $A_nZ_n(\beta r)$. We use the following accustomed notation:

$$Z_n' \equiv \frac{dZ_n(\beta r)}{d(\beta r)} \tag{24}$$

Now the general terms of the Fourier-Bessel expansion of the $(\vec{A}, \phi)$ field are given as follows:

$$E_{rn} = -\phi' = -\frac{ik\beta}{\mu(\sigma + i\omega\varepsilon)} A_n Z_n'(\beta r), \quad (25a)$$

$$E_{\theta n} = -\frac{in}{r}\phi$$
$$= \frac{nk}{\mu(\sigma + i\omega\varepsilon) r} A_n Z_n(\beta r), \quad (25b)$$

$$E_{zn} = -i\omega A_z + ik\phi$$
$$= -(i\omega + \frac{k^2}{\mu(\sigma + i\omega\varepsilon)}) A_n Z_n(\beta r). \quad (25c)$$

$$H_{rn} = \frac{in}{\mu r} A_n Z_n(\beta r), \quad (25d)$$

$$H_{\theta n} = -\frac{\beta}{\mu} A_n Z_n'(\beta r), \quad (25e)$$

$$H_{zn} = 0. \quad (25f)$$

4.2 $(\vec{A}^*, \phi^*)$ field

The equations for the vector potential $\vec{A}^*$ and for the scalar potential ϕ^* are as follows:

$$A_z^{*''} + \frac{1}{r} A_z^{*'} + (\beta^2 - \frac{n^2}{r^2}) A_z^* = 0, \quad (26)$$

$$\phi^* = -\frac{k}{\omega\mu} A_z^*. \quad (27)$$

The corresponding general terms of the electromagnetic field become

$$E_{rn}^* = \frac{in}{r} A_n^* Z_n(\beta r), \quad (28a)$$

$$E_{\theta n}^* = -\beta A_n^* Z_n'(\beta r), \quad (28b)$$

$$E_{zn}^* = 0, \quad (28c)$$

$$H_{rn}^* = -\phi^{*'} = \frac{\beta k}{\omega\mu} A_n^* Z_n'(\beta r), \quad (28d)$$

$$H_{\theta n}^* = -\frac{in}{r}\phi^* = \frac{ink}{\omega\mu r} A_n^* Z_n(\beta r), \quad (28e)$$

$$H_{zn}^* = (\sigma + i\omega\varepsilon) A_n^* + ik\phi^*$$
$$= (\sigma + i\omega\varepsilon - \frac{ik^2}{\omega\mu}) A_n^* Z_n(\beta r). \quad (28f)$$

5 Harmonic field

Let R_o and R_i be, respectively, the outer and inner radii of the tube. For both $R_o < r$ and $r < R_i$ the space is nonconductive, while in the region $R_i < r < R_o$ the space is conductive.

5.1 Non-conductive space

Since $\sigma = 0$ in the non-conductive region,

$$\beta_{vac}^2 = -k^2 + \omega^2\mu\varepsilon. \quad (29)$$

Therefore, the general terms of the $(\vec{A}, \phi)$ field become

$$E_{rn} = -\phi' = -\frac{k\beta_{vac}}{\mu\omega\varepsilon} A_n Z_n'(\beta_{vac} r), \quad (30a)$$

$$E_{\theta n} = -\frac{in}{r}\phi = \frac{nk}{i\mu\omega\varepsilon r} A_n Z_n(\beta_{vac} r), \quad (30b)$$

$$E_{zn} = -i\omega A_z + ik\phi$$
$$= -i(\omega - \frac{k^2}{\mu\omega\varepsilon}) A_n Z_n(\beta_{vac} r). \quad (30c)$$

$$H_{rn} = \frac{in}{\mu r} A_n Z_n(\beta_{vac} r), \quad (30d)$$

$$H_{\theta n} = -\frac{\beta_{vac}}{\mu} A_n Z_n'(\beta_{vac} r), \quad (30e)$$

$$H_{zn} = 0. \quad (30f)$$

Similarly the general terms of the $(\vec{A}^*, \phi^*)$ field are

$$E_{rn}^* = \frac{in}{r} A_n^* Z_n(\beta_{vac} r), \quad (31a)$$

$$E_{\theta n}^* = -\beta_{vac} A_n^* Z_n'(\beta_{vac} r), \quad (31b)$$

$$E_{zn}^* = 0, \quad (31c)$$

$$H_{rn}^* = -\phi^{*'} = \frac{\beta_{vac} k}{\omega\mu} A_n^* Z_n'(\beta_{vac} r), \quad (31d)$$

$$H_{\theta n}^* = -\frac{in}{r}\phi^* = \frac{ink}{\omega\mu r} A_n^* Z_n(\beta_{vac} r), \quad (31e)$$

$$H_{zn}^* = i\omega\varepsilon A_n^* + ik\phi^*$$
$$= i(\omega\varepsilon - \frac{k^2}{\omega\mu}) A_n^* Z_n(\beta_{vac} r). \quad (31f)$$

5.2 Conductive region — inside the tube wall

Inside the metallic tube wall, the displacement current term $i\omega\varepsilon$ can be neglected compared with the conduction term σ. Then,

$$\beta_{met}^2 = -k^2 - i\omega\mu\sigma. \quad (32)$$

The general terms of the $(\vec{A}, \phi)$ field become as follows:

$$E_{rn} = -\phi' = -\frac{ik\beta_{met}}{\mu\sigma} A_n Z_n'(\beta_{met} r), \quad (33a)$$

$$E_{\theta n} = -\frac{in}{r}\phi = \frac{nk}{\mu\sigma r} A_n Z_n(\beta_{met} r), \quad (33b)$$

$$E_{zn} = -i\omega A_z + ik\phi$$
$$= -(i\omega + \frac{k^2}{\mu\sigma})A_n Z_n(\beta_{met}r), \qquad (33c)$$
$$H_{rn} = \frac{in}{\mu r}A_n Z_n(\beta_{met}r), \qquad (33d)$$
$$H_{\theta n} = -\frac{\beta_{met}}{\mu}A_n Z_n'(\beta_{met}r), \qquad (33e)$$
$$H_{zn} = 0. \qquad (33f)$$

Similarly the $(\vec{A}^*, \phi^*)$ field components are

$$E_{rn}^* = \frac{in}{r}A_n^* Z_n(\beta_{met}r), \qquad (34a)$$
$$E_{\theta n}^* = -\beta_{met}A_n^* Z_n'(\beta_{met}r), \qquad (34b)$$
$$E_{zn}^* = 0, \qquad (34c)$$
$$H_{rn}^* = -\phi^{*\prime} = \frac{\beta_{met}k}{\omega\mu}A_n^* Z_n'(\beta_{met}r), \qquad (34d)$$
$$H_{\theta n}^* = -\frac{in}{r}\phi^* = \frac{ink}{\omega\mu r}A_n^* Z_n(\beta_{met}r), \qquad (34e)$$
$$H_{zn}^* = \sigma A_z^* + ik\phi^*$$
$$= (\sigma - \frac{ik^2}{\omega\mu})A_n^* Z_n(\beta_{met}r). \qquad (34f)$$

6 Heuristic study and conclusion

The $(\vec{A}, \phi)$ type electromagnetic field satisfies both conditions 1 and 2 simultaneously, if $n \neq 0$. In the case that $n = 0$, the radial component of magnetic field H_r^{sec} is missed. Thus it is necessary to break the cylindrical symmetry around the tube axis by being equipped with several ferrite cores, as shown in Fig.2. Now we will examine the ratio of E_θ to E_z:

$$\left|\frac{E_\theta}{E_z}\right| = \frac{nk}{\left|i\omega\mu\sigma + k^2\right| r}. \qquad (35)$$

We assume a set of typical values:

- diameter of the probe ≈ 18mm
- height of the probe ≈ 10mm
- electric conductivity of inconel ≈ 1×10^6 mho/m
- Fourier transform variable $k \approx 1$ (see Appendix)
- frequency of exciting current ≈ 400kHz

The ratio is as small as $4n \times 10^{-3}$, therefore the axial component of the electric field is dominant.

On the other hand, the $(\vec{A}^*, \phi^*)$ type electromagnetic field satisfies condition 2. Consequently this type of electromagnetic field can detect axial defects in the tube wall.

As described above, the general terms of Fourier-Bessel expansion of cylindrical electromagnetic field automatically satisfy the first principle of the KM probe, which was first proposed by the Lord Marshall of Goring, provided that the $(\vec{A}, \phi)$ type and $(\vec{A}^*, \phi^*)$ type are simultaneously excited.

Further study of the eddy current probe, e.g. optimization of the design, belongs to computational physics and we must rely upon computer simulation.

Appendix

For the purpose of acquiring a concept of representative value of the variable k, let us compare a Lorentzian curve

$$f(z) = \frac{a^2}{z^2 + a^2} \qquad (36)$$

with its Fourier transform

$$F(k) = ae^{-a|k|}. \qquad (37)$$

It is easy to see that the half-width of the Lorentzian curve is "a" while the half-breadth of the Fourier transform is approximately equal to the reciprocal "$1/a$". Consequently, it is natural to assume the representative value of k is the reciprocal of the length of the probe.